



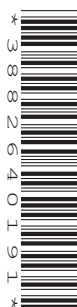
Cambridge International AS & A Level

CANDIDATE
NAME
CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.





1 Find the exact value of $\int_2^{\frac{7}{2}} \frac{1}{\sqrt{4x-x^2-1}} dx$.

[5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

$$x = \cosh t, \quad y = \sinh t, \quad \text{for } 0 < t \leq \frac{3}{5}.$$

(a) Show that $s = \int_0^{\frac{3}{5}} \sqrt{\cosh 2t} \, dt$.

[4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



5

[illegible]



3 The curve C has equation

$$x^3 + 2xy + 8y^3 = -12.$$

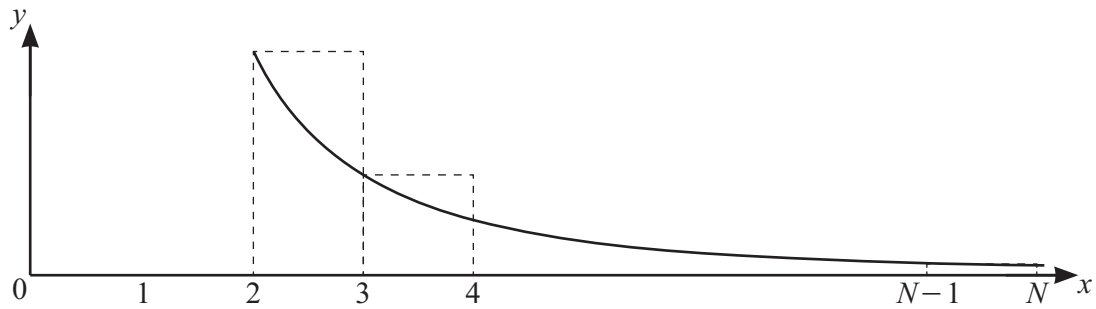
(a) Show that, at the point $(-2, -1)$ on C , $\frac{dy}{dx} = -\frac{1}{2}$. [3]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



[5]

[illegible]



The diagram shows the curve with equation $y = x^{-2}$ for $2 \leq x \leq N$ together with a set of $(N-2)$ rectangles of unit width.

- (a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{1}{r^2} > \frac{3}{2} - \frac{1}{N} + \frac{1}{N^2}. \quad [5]$$

[illegible]

[illegible]

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.



5 (a) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 338 \sin t. \quad [7]$$

[illegible]



9

where the constants R and ϕ are to be determined. [3]

where the constants R and ϕ are to be determined. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.



- 6 (a) Show that $\sum_{r=1}^n z^{4r} = \frac{z^{4n+2} - z^2}{z^2 - z^{-2}}$, for $z^2 \neq z^{-2}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) By letting $z = \cos \theta + i \sin \theta$, show that, if $\sin 2\theta \neq 0$,

$$\sum_{r=1}^n \sin(4r\theta) = \frac{\cos 2\theta - \cos(4n+2)\theta}{2 \sin 2\theta}. \quad [5]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



[illegible]



7 (a) Show that

$$\frac{d}{dx} \left(\frac{x}{2} \sqrt{x^2 - 9} - \frac{9}{2} \cosh^{-1} \frac{x}{3} \right) = \sqrt{x^2 - 9}. \quad [3]$$

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \sqrt{x^2 - 9},$$

given that $y = 1$ when $x = 3$. Give your answer in the form $y = f(x)$. [9]



[illegible]

- (a) Find Cartesian equations of Π_1 and Π_2 . [3]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

- (b) Express the vector equation of l in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{a} + \lambda \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are vectors to be determined, and hence show that for points on l , $\frac{1}{2}x + \frac{1}{12}y = 1$ and $z = 0$. [2]

[illegible]



DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

- DO NOT WRITE IN THIS MARGIN

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]



1

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

